LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

B.Sc. DEGREE EXAMINATION – **STATISTICS**

THIRD SEMESTER – NOVEMBER 2007

ST 3500 - STATISTICAL MATHEMATICS - II

BB 23

Date : 27/10/2007 Dept. No. Max. : 100 Marks

Time : 9:00 - 12:00

PART – A

(10 x 2 = 20 marks)

Answer ALL questions.

1. Define upper integral and lower integral of a function f.
2. Let f be the function defined on [0,1] by

$$f(x)=\begin{cases}0, & when\ x\ is\ irrational\\ 1, & when\ x\ is\ rational\end{cases}$$

check whether the function f is Riemann integrable on [0,1].

3. Find $\frac{\partial u}{\partial x}$ and $\frac{\partial u}{\partial x}$ if u = $\frac{xy}{x+y}$
4. A continuous random variable X has the p.d.f given by

$$f(x)=\begin{cases}k\,x e^{-\lambda x}, & x>0,\ \lambda>0\\ 0, & otherwise\end{cases}$$

Determine the constant k.

5. Solve the equation $(D^2+1)y=0$
6. Find the order and degree of the differential equation

$$\frac{d^2y}{dx^2}-4\sqrt{\frac{dy}{dx}}=0$$

7. Define Beta distribution of second kind.
8. Find the mean of Gamma distribution with parameter λ.
9. Find the rank of the matrix $\begin{bmatrix}1 & 2\\ 2 & 4\end{bmatrix}$
10. Define the rank of a matrix.

PART – B

(5 x 8 = 40 marks)

Answer any FIVE questions.

11. For each $n\in I$, let σ_n be the sub division $\left\{0,\ \frac{1}{n},\ \frac{2}{n},....,\frac{n}{n}\right\}$ of [0,1].

Compute $\underset{n\to\infty}{\ell t}\, L[f:\sigma_n]$ and $\underset{n\to\infty}{\ell t}\, U[f:\sigma_n]$ for the function $f(x)=x^2,\ 0\le x\le 1$.

12. The joint p.d.f of a 2 dimensional random variable (x,y) is given by:

$$f(x,y)=\begin{cases}2, & 0<x<1,\ 0<y<x\\ 0, & otherwise\end{cases}$$

Find the marginal p.d.fs of X and Y.

13. Express $\begin{bmatrix}6 & 8 & 5\\ 4 & 2 & 3\\ 9 & 7 & 1\end{bmatrix}$ as the sum of a symmetric and a skew symmetric matrix.
14. Solve the following differential equation $(D^2-2D+1)y+x^2+1+Sin2x$.

15. Find the minimum of the function

$$f(x,y) = 3x^2 + 4y^2 - xy \text{ if } 2x + y = 21.$$

16. The joint p.d.f of a two dimensional random variable (X,Y) is

$$f(x,y) + \begin{cases} 4xy\ e^{-(x^2+y^2)} & , \quad x,y \geq 0 \\ 0 & , \quad otherwise \end{cases}$$

Prove that the p.d.f of $u = \sqrt{x^2 + y^2}$ is

$$h(u) = \begin{cases} 2u^3 e^{-u^3} & , \ u \geq 0 \\ 0 & , \ otherwise \end{cases}$$

17. Find mean and variance of Beta distribution of 1st kind with parameters μ, α.
18. State and prove additive property of Gamma distribution.

PART – C

(2 x 20 = 40 marks)

Answer any TWO questions.

19. a) Find the moment generating function of the random variable, whole moments are

$\mu^1_{\wp}=(r+1)!3^{\tau}, \qquad for\ \tau=0,1,2,..........$

b) Evaluate $\int_0^{\pi/2} \frac{\sqrt{Sinx}}{\sqrt{Sinx} + \sqrt{Cosx}}\, dx$

20. a) If X is a continuous random variable with p.d.f

$$f(x) = \begin{cases} 2(1-x), & 0 < x < 1 \\ 0 \quad , & otherwise \end{cases}$$

find M.G.F of X and hence find β_1 and β_2.

b) Let X be a continuous random variable with p.d.f

$$f(x) = \begin{cases} \frac{1}{4} \ , & 1 < x < 5 \\ 0 \ , & otherwise \end{cases}$$

prove that $E[x(x+7)] \neq E(x)E(x+7)$

21. a) The joint probability distribution of (x,y) is given by

$$f(x,y) = \begin{cases} \frac{x+y}{21}, & x = 1,2,3 \quad and y = 1,2 \\ 0 \ , & otherwise \end{cases}$$

Examine whether X and Y are independent.

b) Solve the following differential equations

(i) $(D^2 - 6D + 9)y = e^{3x}$

(ii) $(D^2 + 4)y = Sin 3x$

22. a) Find the characteristic roots of the matrix $\begin{bmatrix} 0 & 1 & 2 \\ 1 & 0 & -1 \\ 2 & -1 & 0 \end{bmatrix}$

b) Show that the following system of equations is consistent and solve them.

$x - 3y - 8z = -10; \qquad 3x + y - 4z = 0;$

$2x + 5y + 6z = 13$

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